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PHASE STRUCTURE OF CAUSAL DYNAMICAL TRIANGULATIONS IN 4D*

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Causal Dynamical Triangulations (CDT) is a lattice approach to quantum gravity. CDT has rich phase structure, including a semiclassical phase consistent with Einstein's general relativity. Some of the observed phase transitions are second (or higher) order which opens a possibility of investigating the ultraviolet continuum limit. Recently, a new phase with intriguing geometric properties has been discovered and the new phase transition is also second (or higher) order.

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1. Introduction

Quantum field theory (QFT) techniques provide powerful tools in describing three out of four fundamental interactions. The key problem in applying these methods to quantize gravity is that QFT based on Einstein's general relativity (GR) is perturbatively nonrenormalizable [1]. However, following Weinberg's asymptotic safety conjecture [2], it is possible that in the space of gravitational couplings, there exist non-Gaussian ultraviolet (UV) fixed point(s) at which nonperturbative QFT techniques could be used to define a quantum theory of gravity valid for any energy scale[1]. Among such techniques, lattice methods play an increasingly important role. A good test of any lattice approach is its ability to reproduce GR in the infrared limit and also the existence of second (or higher) order phase transitions

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[1] There are known examples of perturbatively nonrenormalizable but asymptotically safe QFTs and evidence is growing that it is also the case for gravity [3].

associated with the perspective UV fixed point(s)[2]. Therefore, a study of the phase structure and the order of phase transitions constitute first steps in the quest for the continuum theory of quantum gravity.

2. Causal Dynamical Triangulations

Causal Dynamical Triangulations (CDT) is a lattice model based on the path integral quantization applied to GR. CDT gives a precise meaning to a (formal) gravitational path integral

$$\mathcal{Z}_{\mathrm{GR}} = \int \mathcal{D}_{\mathcal{M}}[g] e^{iS_{\mathrm{HE}}[g]} \quad \rightarrow \quad \mathcal{Z}_{\mathrm{CDT}} = \sum_{\mathcal{T}} e^{iS_{\mathrm{R}}[\mathcal{T}]}\,, \tag{1}$$

approximating continuous geometries (described by all physically distinct metric tensors g) by lattices constructed from two types of identical four-dimensional simplicial blocks glued together to form triangulations $\mathcal{T}$. The key assumption of CDT is an introduction of causal structure by foliating spacetime into Cauchy hyper-surfaces Σ of constant global proper time T. Topologically, a triangulation $\mathcal{T}$ is $\Sigma \times T$, and one requires that the topology of each spatial slice Σ is fixed. Each spatial layer Σ at integer (lattice) time t is by definition constructed from equilateral tetrahedra. The four-simplices interpolate between consecutive spatial layers of integer t in such a way that also all intermediate Cauchy layers between t and $t+1$ have the requested fixed spatial topology. This can be done by using just two types of four-simplices called the $(4,1)$ simplex and the $(3,2)$ simplex[3]. It is assumed that the interior of each four-simplex is a flat Minkowski spacetime and local curvature is defined by the way the simplices are glued together. In Eq. (1), S_{R} is the discretized Hilbert–Einstein action S_{HE} obtained following Regge's method for describing piecewise linear geometries [4]

$$S_{\mathrm{R}} = -\left(\kappa_0 + 6\Delta\right) N_0 + \kappa_4 \left(N_{(4,1)} + N_{(3,2)}\right) + \Delta\, N_{(4,1)}\,, \tag{2}$$

where $N_{(4,1)}$, $N_{(3,2)}$ and N_0 denote the total number of $(4,1)$ simplices, $(3,2)$ simplices and vertices, respectively, while κ_0, Δ and κ_4 are three bare coupling constants. They are functions of the Newton's constant, the cosmological constant and the asymmetry α between lengths of time-like and space-like links in the lattice ($a_t^2 = -\alpha\, a_s^2$).

In order to study the regularised path integral, one is forced to use Monte Carlo techniques. This can be done by applying the Wick rotation from positive to negative α values which changes time-like links into space-like

[2] Infinite correlation lengths characteristic of such phase transitions make it, in principle, possible to decrease lattice spacing to zero, *i.e.* to investigate the continuum limit.

[3] The numbers in parentheses denote vertices lying in t and $t \pm 1$, respectively.

links, *i.e.* changes real (Lorentzian) time $t^{(\mathrm{L})}$ into imaginary (Euclidean) time $t^{(\mathrm{E})}$ ($t^{(\mathrm{L})} \to t^{(\mathrm{E})} = it^{(\mathrm{L})}$), and also changes Lorentzian action into Euclidean action ($S_{\mathrm{R}}^{(\mathrm{L})} \to S_{\mathrm{R}}^{(\mathrm{E})} = iS_{\mathrm{R}}^{(\mathrm{L})}$). Accordingly, the path integral $\mathcal{Z}_{\mathrm{CDT}}$ (1) becomes a partition function which can be studied numerically.

3. Phase structure of 4-dim CDT

All results presented herein were obtained for a particular choice of fixed spatial topology $\Sigma = S^3$ (3-sphere)[4]. Historically, three phases of various spacetime geometry called A, B and C were discovered (see Fig. 1). Phase A (time-uncorrelated geometry) and B (time-collapsed geometry) do not have clear physical interpretation. The most interesting one is phase C, which is separated from phase A by a 1st order transition and from phase B by a 2nd (or higher) order transition [5]. The basic feature of phase C (now C_{dS}) is the emergence of large scale four-dimensional geometry [6] consistent with semiclassical (Euclidean) de Sitter universe [7]. It was also shown that in this phase, quantum fluctuations of spatial volume are governed by the minisuperspace reduction of the Hilbert–Einstein action [7, 8].

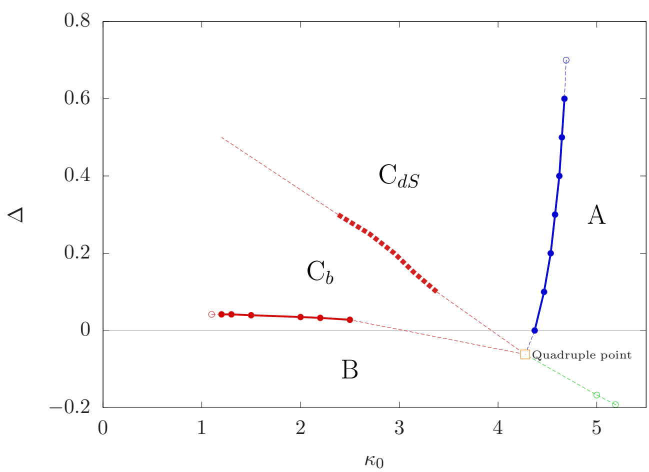

Fig. 1. Phase structure of 4-dim CDT in the (κ_0, Δ) bare couplings plane. κ_4 is fine-tuned to critical value consistent with infinite volume limit.

The key tool in the analysis of the effective action of CDT was the transfer matrix (TM) parametrised by a spatial volume observable, *i.e.* the transition amplitude from spatial volume n at (lattice) time t to spatial volume m at time $t + 1$. Deep inside phase C (C_{dS} region in Fig. 1), the

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[4] Preliminary studies of CDT with toroidal spatial topology confirm the existence of similar phase structure.

TM is well-parametrised by [9]

$$\langle n|M_{C_{\mathrm{dS}}}|m\rangle = \underbrace{\exp\left[-\frac{1}{\mathit{\Gamma}}\frac{(n-m)^2}{(n+m)}\right]}_{\text{kinetic part}}\underbrace{\exp\left[-\mu\left(\frac{n+m}{2}\right)^{1/3}+\lambda\left(\frac{n+m}{2}\right)\right]}_{\text{potential part}}, \tag{3}$$

where $\mathit{\Gamma}$, μ and λ are parameters related to the Newton constant, the size of the CDT universe and the cosmological constant, respectively. However, it was observed that in some region of the parameter space close to phase A (C_{b} region in Fig. 1), the TM kinetic part bifurcates, such that [10]

$$\begin{aligned}\langle n|M_{C_{\mathrm{b}}}|m\rangle = &\left[\exp\left(-\frac{1}{\mathit{\Gamma}}\frac{\left((n-m)-[c_0(n+m-s_{\mathrm{b}})]_+\right)^2}{n+m}\right)\right.\\ &+\left.\exp\left(-\frac{1}{\mathit{\Gamma}}\frac{\left((n-m)+[c_0(n+m-s_{\mathrm{b}})]_+\right)^2}{n+m}\right)\right]\times\text{potential part}[n+m]\,,\end{aligned} \tag{4}$$

which led to a discovery of a new phase transition associated with the parameters $s_{\mathrm{b}}\to\infty$ and $c_0\to 0$, where TM (4) transforms into TM (3).

The newly discovered bifurcation phase C_{b} has many intriguing geometric properties, including very large (potentially infinite) Hausdorff dimension and also spectral dimension becoming much larger than 4 for long diffusion times. The C_{dS}–C_{b} phase transition is related to breaking of spatial homogeneity of phase C_{dS} by the appearance of compact spatial volume clusters concentrated around "singular" vertices with macroscopically large coordination number present every second time layer inside phase C_{b} [11]. The volume clusters have (topologically) spherical boundaries and thus can technically be called "black balls". The "black balls" around singular vertices in time t and $t+2$ are causally connected through those intermediate 4-simplices which also share one of the "singular" vertices. As a result, one observes a marked-out four-dimensional geometric structure related to evolution of time-correlated "black ball" volume condensations. The geometry of phase C_{b} requires further studies, but a working hypothesis is that what is observed might be actually a quantum black hole.

A key question remains about the order of the recently discovered bifurcation transition. In Ref. [12], an order parameter related to the appearance of high order vertices was proposed,

$$\mathrm{OP}_2 = \tfrac{1}{2}\left[|O_{\max}(t_0) - O_{\max}(t_0+1)| + |O_{\max}(t_0) - O_{\max}(t_0-1)|\right], \tag{5}$$

where $O_{\max}(t_0)$ is the highest coordination number among all vertices and $O_{\max}(t_0\pm 1)$ is the highest coordination number of a vertex observed in the neighbouring time slice. The position of the $C_{\rm dS}$–$C_{\rm b}$ transition is signalled by a peak in susceptibility $\chi_{\rm OP_2} = \langle {\rm OP}_2 \rangle - \langle {\rm OP}_2 \rangle^2$ and it moves in the CDT bare couplings space (κ_0, Δ) when lattice volume $N_{(4,1)}$ is changed. The volume dependence of critical Δ (for $\kappa_0 = 2.2$ fixed) can be fitted with the following function:

$$\Delta^{\rm crit}\left(N_{(4,1)}\right) = \Delta^{\rm crit}(\infty) - \alpha\, N_{(4,1)}^{-1/\gamma}\,, \tag{6}$$

and measured value of the critical exponent $\gamma = 2.71 \pm 0.34$ strongly supports the conjecture that the bifurcation transition is a 2$^{\rm nd}$ (or higher) order phase transition[5].

4. Summary and conclusions

We have briefly presented the recently updated 4-dim CDT phase diagram, including the semiclassical phase $C_{\rm dS}$ which seems to be CDT realisation of the correspondence principle, and the newly discovered bifurcation phase $C_{\rm b}$ with very nontrivial geometric properties. We have provided evidence that the $C_{\rm dS}$–$C_{\rm b}$ phase transition, related to breaking of homogeneity of semiclassical phase, is a 2$^{\rm nd}$ (or higher) order phase transition which may in principle allow one to approach the perspective UV fixed point of quantum gravity [13].

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